Quiz 3

COL 352
Introduction to Automata & Theory of Computation

Problem 1

Let $L_H = \{ < M, w > \mid M \text{ halts on input } w\}$. Is L_H recursive, or r.e. but not recursive or not r.e. ? Justify

Solution : L_H is r.e. but not recursive

Proof :

I) Recursively enumerable : we can construct a Turing Machine M' which semi-decides L_H -

1. For any input M and w, M' simulates M on w.
2. If M halts on input w, go to final state in M'.

Hence L_H is recursively enumerable.

II) Not recursive : we know that Universal TM language is an undecidable/non-recursive language.

We can show that L_H is also non-recursive by reducing the Universal TM language (L_U) to L_H, i.e. $L_U \leq_f L_H$, where f is a Turing computable function and is defined as $f(< M,\ w >) = < M',\ w' >$ such that M' halts on w iff M accepts w, i.e.

1. First simulate M on w.
2. If M accepts w (i.e. it stops in an accepting state), M' should accept w'.
3. If M doesn't stop on w then let M' keep moving right (infinite loop).

Hence this reduction shows that L_H is not recursive.

Problem 2

A student unconvinced by the diagonalisation argument for proving L_d is not e.e., approaches her Professor with the following doubt. Since the set L_d is dependent on the ordering of the strings, what if, a different ordering $\mathcal{O}'$ is used? Why will the previous L_d still continue to be a non r.e. set although it does not correspond to the diagonal in $\mathcal{O}'$? Can you answer her doubts? You can assume that both orderings can be computed using a TM.

Solution : Given L_d is NOT recursively enumerable set which corresponds to the diagonal in ordering $\mathcal{O}$. Let L'_d be the NOT recursively enumerable set which corresponds to the diagonal in new ordering $\mathcal{O}'$.

Also let old set L_d, which does not correspond to the diagonal in the ordering $\mathcal{O}'$, be represented as S. Then we have to show that S is still NOT recursively enumerable set although it does not correspond to the diagonal in $\mathcal{O}'$. We will prove the same by using reducibility.

Given that both orderings can be computed using a TM. So the identity map $f : L_d \to S$ is computable which is defined by $f(w_i) = w'_j$ where w_i is ith string in ordering $\mathcal{O}$ and w'_j is jth string in ordering $\mathcal{O}'$ such that $w_i = w'_j$. Since this identity map is a bijection, so we have $L_d \leq_f S$. If we assume that S is recursively enumerable, then $L_d \leq_f S$ implies that L_d is also recursively enumerable which is a contradiction.

Hence S must be NOT recursively enumerable which implies that L_d remains NOT recursively enumerable set although it does not correspond to the diagonal in $\mathcal{O}'$.